Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Sample Assessment Materials for first teaching September 2018

(Time: 1 hour 30 minutes)	Paper Reference **WMA11/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P1

You must have: Mathematical Formulae and Statistical Tables, calculator	Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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Answer ALL questions. Write your answers in the spaces provided.

1. Given that $y = 4x^3 - \dfrac{5}{x^2}$, $x \neq 0$, find in their simplest form

(a) $\dfrac{dy}{dx}$,

(3)

(b) $\int y \, dx$

(3)

a) ① rewrite equation in easier form for integration

$y = 4x^3 - \dfrac{5}{x^2} = 4x^3 - 5x^{-2}$ * indicies rule: $\dfrac{a}{x^b} = ax^{-b}$

② differentiate (dy/dx is differential of y)

$dy/dx = 3(4x^{3-1}) + (-2)(-5x^{-2-1}) = 12x^2 + 10x^{-3}$

$\therefore dy/dx = 12x^2 + 10x^{-3}$

b) $y = 4x^3 - 5x^{-2}$

integration

$\int y \, dx = \int 4x^3 - 5x^{-2} \, dx = \left[\left(\frac{4}{3+1}x^{3+1}\right) + \left(\frac{-5}{-2+1}x^{-2+1}\right)\right] = 1x^4 + 5x^{-1} + C$

DON'T FORGET or will lose a mark

$\therefore \int y \, dx = x^4 + 5x^{-1} + c$

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Question 1 continued

Q1

(Total for Question 1 is 6 marks)

2. (a) Given that $3^{-1.5} = a\sqrt{3}$ find the exact value of a

(2)

(b) Simplify fully $\dfrac{(2x^{\frac{1}{2}})^3}{4x^2}$

(3)

a) $3^{-1.5} = a\sqrt{3} \rightarrow 3^{-\frac{3}{2}} = a\sqrt{3}$

$3^{-\frac{3}{2}} = a\sqrt{3} \rightarrow \dfrac{1}{3^{3/2}} = a\sqrt{3}$ ① ① indicies rule : $a^{-b} = \dfrac{1}{a^b}$

$\dfrac{1}{3^{3/2}} = a\sqrt{3} \rightarrow \dfrac{1}{\sqrt[2]{3^3}} = a\sqrt{3}$ ② ② indicies rule : $a^{\frac{b}{c}} = \sqrt[c]{a^b}$

$\dfrac{1}{\sqrt{3^3}} = a\sqrt{3} \rightarrow \dfrac{1}{\sqrt{3^{2+1}}} = a\sqrt{3} \rightarrow \dfrac{1}{\sqrt{3^2 \times 3^1}} = a\sqrt{3}$ ③

③ $a^{b+c} = a^b \times a^c$

$\dfrac{1}{\sqrt{3^2}\sqrt{3}} = a\sqrt{3} \rightarrow \dfrac{1}{3\sqrt{3}} = a\sqrt{3}$

rationalising denominator :

$\dfrac{1}{3\sqrt{3}} \dfrac{\times 3\sqrt{3}}{\times 3\sqrt{3}} = \dfrac{1(3\sqrt{3})}{(3\sqrt{3})(3\sqrt{3})} = \dfrac{3\sqrt{3}}{27} = \dfrac{3}{27}\sqrt{3} = \dfrac{1}{9}\sqrt{3}$

$\dfrac{1}{9}\sqrt{3} = a\sqrt{3}$

$\therefore a = \dfrac{1}{9}$

b) $\dfrac{(2x^{\frac{1}{2}})^3}{4x^2} = \dfrac{2^3 x^{\frac{1}{2}\times 3}}{4x^2}$ ④ ④ indicies rule $a^{bc} = (a^b)^c = (a^c)^b$

$\dfrac{2^3 x^{\frac{3}{2}}}{4x^2} = \dfrac{8x^{3/2}}{4x^2} = \dfrac{8}{4} \times \dfrac{x^{3/2}}{x^2} = 2\,\dfrac{x^{3/2}}{x^2}$

$2\,\dfrac{x^{3/2}}{x^2} = 2\,x^{3/2-2} = 2x^{-1/2}$ ⑤ ⑤ indicies rule : $\dfrac{a^b}{a^c} = a^{b-c}$

Leave blank

Question 2 continued

$\therefore\ 2x^{-1/2} = \dfrac{2}{\sqrt{x}}$

Q2

(Total for Question 2 is 5 marks)

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3. Solve the simultaneous equations

$$y + 4x + 1 = 0$$

$$y^2 + 5x^2 + 2x = 0$$

(6)

① rearrange $y+4x+1=0$ to make y the subject.

$$y+4x+1=0$$
$$-1 \qquad y+4x=-1 \qquad -1$$
$$-4x \qquad y=-4x-1 \qquad -4x$$

② Substitute $y=-4x-1$ into equation $y^2+5x^2+2x=0$ & Simplify.

$$y^2+5x^2+2x=0$$
$$(-4x-1)^2+5x^2+2x=0$$
$$(-4x-1)(-4x-1)+5x^2+2x=0$$
$$(16x^2+8x+1)+5x^2+2x=0$$
$$21x^2+10x+1=0$$

③ Solve quadratic equation $21x^2+10x+1=0$

factorise: $(7x+1)(3x+1)=0$

Solve: • $7x+1=0$ • $3x+1=0$

$7x=-1$ $\qquad 3x=-1$

$x=-1/7$ $\qquad x=-1/3$

④ Substitute $x=-1/7$ & $x=-1/3$ into either one of the Simultaneous equations.

$$y+4x+1=0$$

when $x=-1/7$

$$y+4(-1/7)+1=0$$
$$y-4/7+1=0$$
$$-3/7 \qquad y+3/7=0 \qquad -3/7$$
$$y=-3/7$$

when $x=-1/3$

$$y+4(-1/3)+1=0$$
$$y-4/3+1=0$$
$$+\tfrac{1}{3} \qquad y-1/3=0 \qquad +\tfrac{1}{3}$$
$$y=1/3$$

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DO NOT WRITE IN THIS AREA

Question 3 continued

$\therefore \; x = -\frac{1}{7}, \; y = -\frac{3}{7} \quad \& \quad x = -\frac{1}{3}, \; y = \frac{1}{3}$

Q3

(Total for Question 3 is 6 marks)

4. The straight line with equation $y = 4x + c$, where c is a constant, is a tangent to the curve with equation $y = 2x^2 + 8x + 3$

Calculate the value of c

(5)

tangent means gradient of tangent is same as gradient of curve

in the form $y = mx + c$, m is gradient $\therefore$ for $y = 4x + c$, m is 4.

① To find point on curve where gradient is 4, first find the gradient function (differential) dy/dx.

$$y = 2x^2 + 8x^1 + 3x^0 \rightarrow \because x^0 = 1$$

$$dy/dx = 2(2x^{2-1}) + 1(8x^{1-1}) + 0(3x^{0-1}) = 4x + 8$$

② equate dy/dx to 4, & solve for x to find x-coordinate where gradient of curve is 4.

$$dy/dx = 4x + 8 = 4$$

$$-8 \quad 4x = -4 \quad -8$$

$$\div 4 \quad x = -1 \quad \div 4$$

③ Substitute $x = -1$ into equation of curve to find y-coordinate.

$$y = 2x^2 + 8x + 3$$

$$y = 2(-1)^2 + 8(-1) + 3 = 2 - 8 + 3 = -3$$

④ Find equation of tangent, to find $+c$ using line passing through coordinates (a, b) & has gradient m.

$(-1, -3)$ equation: $(y - b) = m(x - a)$

$a = -1$

$b = -3$

$m = 4$

$$(y - (-3)) = 4(x - (-1))$$

Question 4 continued

5 Write in form $y = mx + c$

$$y + 3 = 4(x + 1)$$

$$-3 \; y + 3 = 4x + 4 \; -3$$

$$y = 4x + 1$$

$$y = 4x + c$$

$$y = 4x + 1$$

$$\therefore \; c = 1$$

Q4

(Total for Question 4 is 5 marks)

5. (a) On the same axes, sketch the graphs of $y = x + 2$ and $y = x^2 - x - 6$ showing the coordinates of all points at which each graph crosses the coordinate axes.

(4)

(b) On your sketch, show, by shading, the region R defined by the inequalities

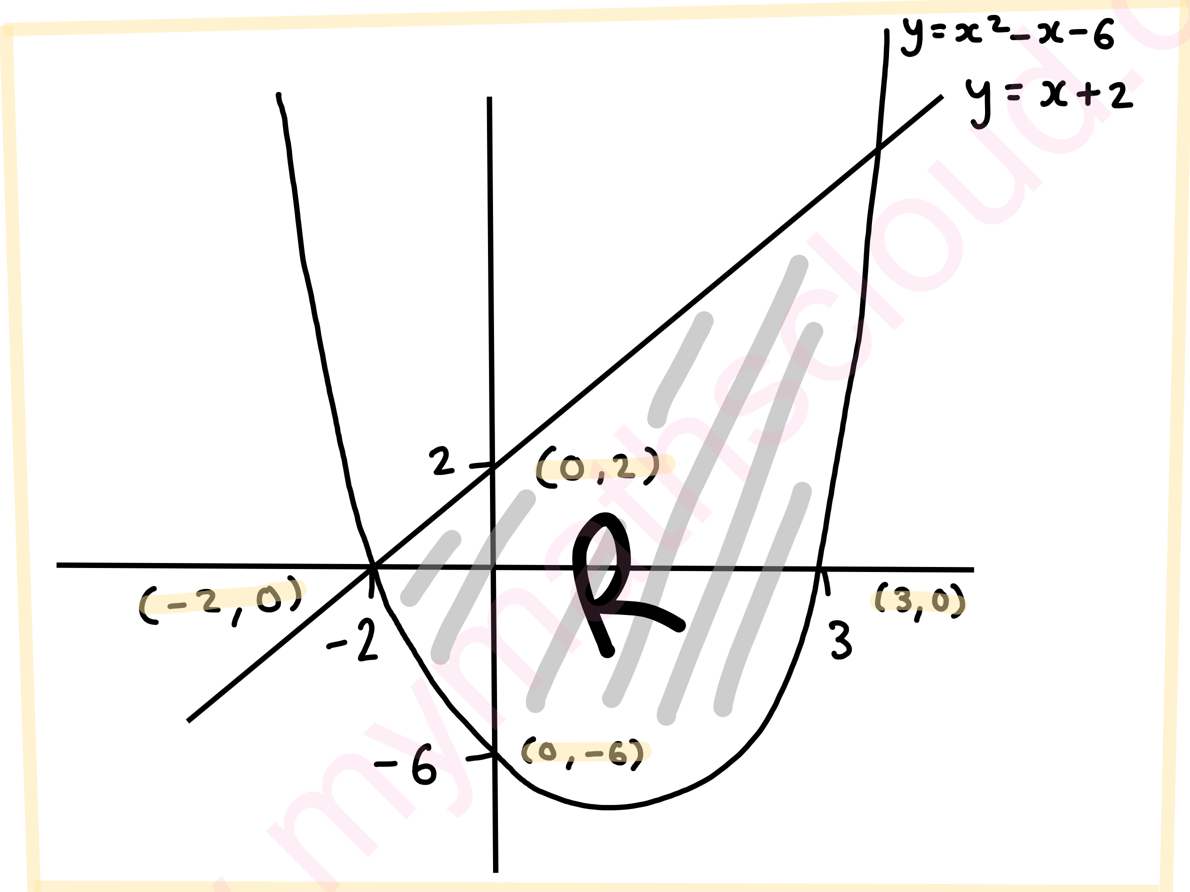
$$y < x + 2 \quad \textbf{and} \quad y > x^2 - x - 6$$

(1)

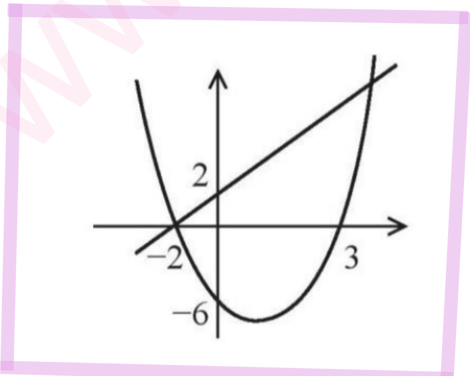
(c) Hence, or otherwise, find the set of values of x for which $x^2 - 2x - 8 < 0$

(3)

a) & b)



a) markscheme



Question 5 continued

a) working out to draw the graphs

① $y = x + 2$

y intercept : when $x = 0$, $\boxed{y = 2}$

x intercept : when $y = 0$, $0 = x + 2 \rightarrow \boxed{x = -2}$

② $y = x^2 - x - 6$

y intercept : when $x = 0$, $\boxed{y = -6}$

x intercepts : when $y = 0$, $x^2 - x - 6 = 0$

factorise : $(x - 3)(x + 2) = 0$

Solve : • $x - 3 = 0$ • $x + 2 = 0$

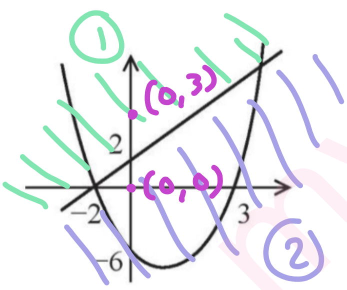
$\boxed{x = 3}$ $\boxed{x = -2}$

Due to presence of $x^{\boxed{2}}$ equation is a quadratic so shape is ∪ or ∩.

x^2 is positive ∴ positive quadratic ∴ Shape is a 'smiley mouth' ∪.

b) Explanation:

① $y < x + 2$



find region where y-value is less than $x+2$.

Either ① or ②

in region 1 : choose coordinate $(0, 3)$

$3 < 0 + 2$

$3 < 0$

Inequality FALSE (3 is NOT less than 2). This is not correct region.

in region 2 : choose coordinate $(0, 0)$

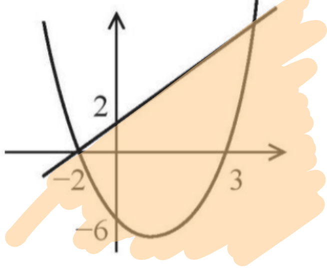
$0 < 0 + 2$

$0 < 2$

Inequality TRUE (0 is less than 2) ∴ this is correct region.

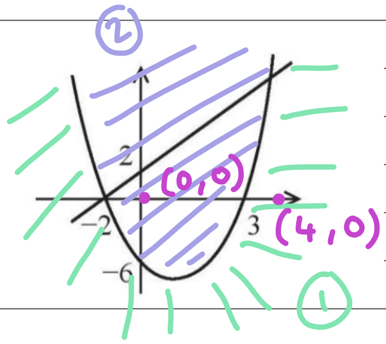
Q5

(Total for Question 5 is 8 marks)



← $y < x+2$

② $y > x^2 - x - 6$



find region where y-value is greater than x^2-x-6.

Either ① or ②

in region ①: choose coordinate (4, 0)

$0 > 4^2 - 4 - 6$

$0 > 6$

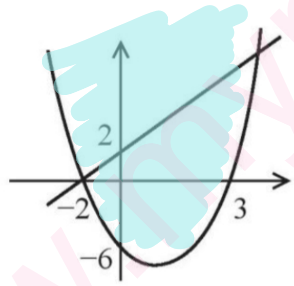
Inequality FALSE (6 is NOT less than 0). This is not correct region.

in region 2: choose coordinate (0, 0)

$0 > 0^2 - 0 - 6$

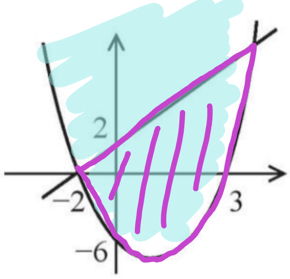
$0 > -6$

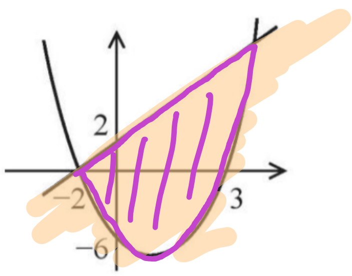
Inequality TRUE (-6 is less than 0) ∴ this is correct region.



← $y > x^2 - x - 6$

③ To find region R, identify overlap in the two shaded areas.





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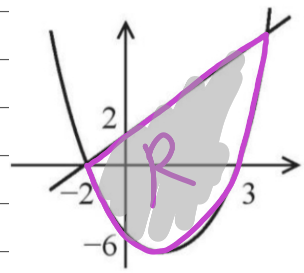


c) when $y < x+2$ & $y > x^2-x-6$

$x^2-x-6 < y < x+2$

$\therefore \quad x^2-x-6 < x+2$

① Simplify :

$$\begin{aligned} x^2-x-6 &< x+2 \\ x^2-2x-6 &< 2 \\ x^2-2x-8 &< 0 \end{aligned}$$

$-x$ (both sides), -2 (both sides)

② factorise $x^2-2x-8<0$

$(x-4)(x+2)<0$

③ Find critical values by solving for x when $(x-4)(x+2)=0$

- $x-4=0$, $x=4$
- $x+2=0$, $x=-2$

when $ax^2+bx+c<0$ & critical values are x_1 & x_2,

inequality is $x_1 < x < x_2$

where $x_1 < x_2$

$\therefore \quad -2 < x < 4$

6.

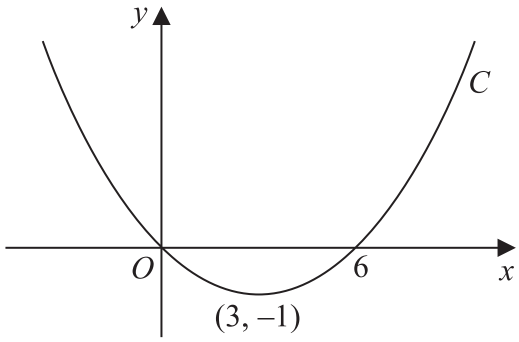


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$

The curve C passes through the origin and through (6, 0)

The curve C has a minimum at the point (3, –1)

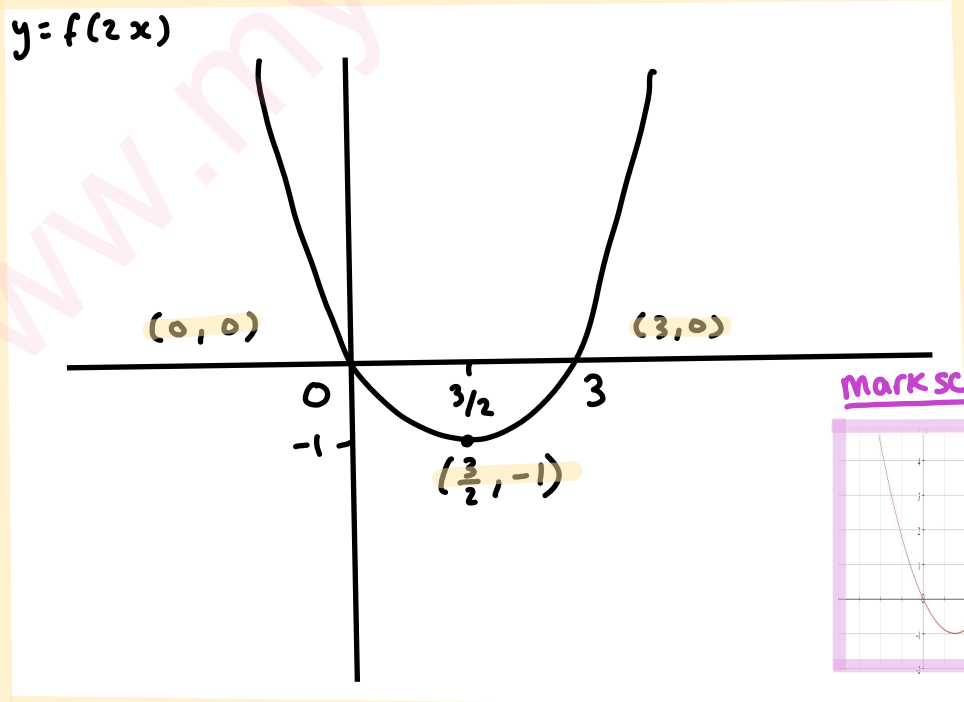
On separate diagrams, sketch the curve with equation

(a) $y = \mathrm{f}(2x)$: Translation is horizontal squash by factor 2, so all x-coordinates multiply by $\frac{1}{2}$.
Y-coordinates not affected ∵ 2 is inside f(x) brackets. **(3)**

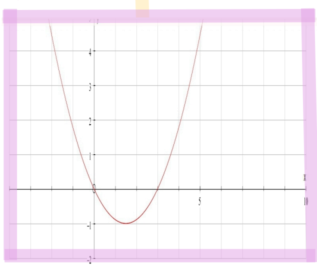
(b) $y = \mathrm{f}(x + p)$, where p is a constant and $0 < p < 3$ **(4)**

On each diagram show the coordinates of any points where the curve intersects the x-axis and of any minimum or maximum points.

a) $(3,-1) \rightarrow (3 \times \frac{1}{2}, -1) \rightarrow (\frac{3}{2}, -1)$ OR $(1.5, -1)$
$(0,0) \rightarrow (0 \times \frac{1}{2}, 0) \rightarrow (0, 0)$
$(6,0) \rightarrow (6 \times \frac{1}{2}, 0) \rightarrow (3, 0)$



Mark scheme:



Question 6 continued

b) Working out: $f(x+p)$ where $0<p<3$

translation is $\begin{pmatrix} p \\ 0 \end{pmatrix}$ → move graph left by p units.

We know direction is left ∵ p is a positive value $(0<p<3)$

So we subtract p from x-coordinates of graph (we do inverse of what is in $f(x)$ brackets).

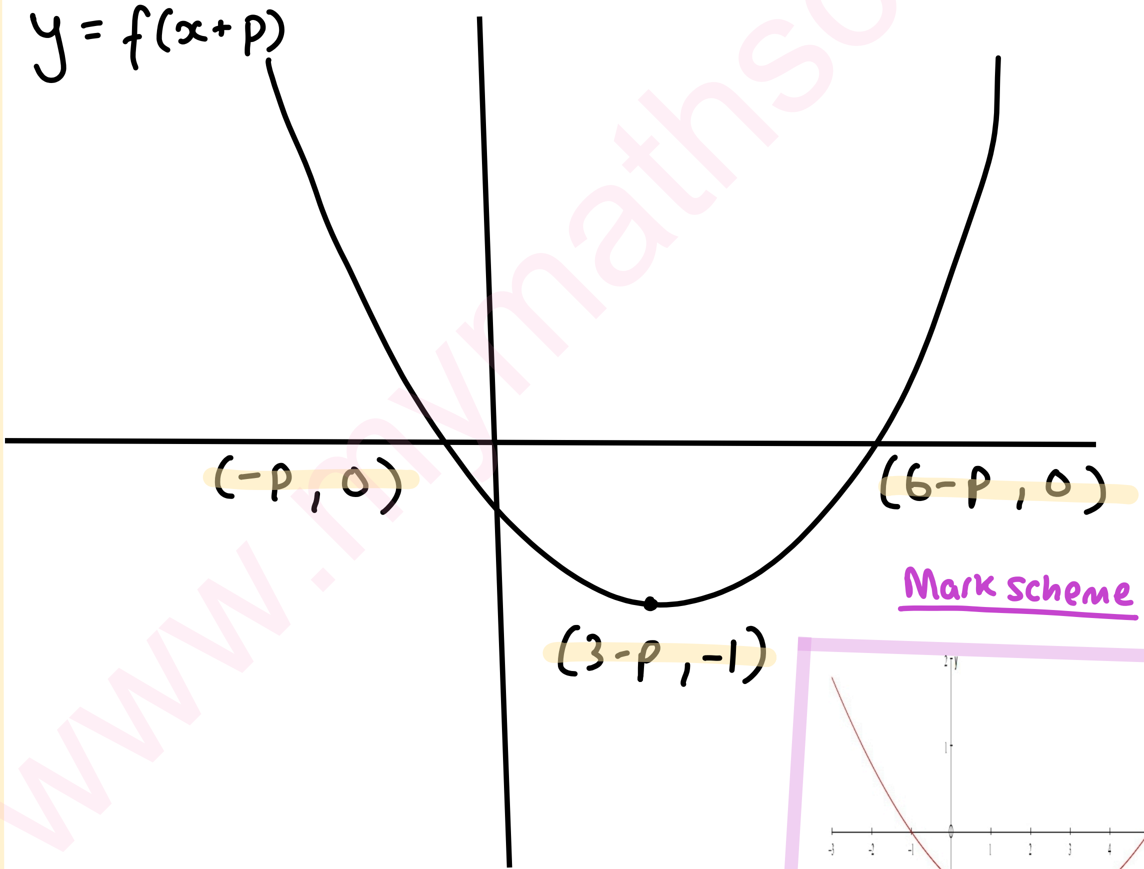
y-coordinates not affected ∵ p is inside $f(x)$ brackets.

$(3,-1) \rightarrow (3-p, -1)$
$(0,0) \rightarrow (0-p, 0)$
$(6,0) \rightarrow (6-p, 0)$

p is less than 3 ∴ curve minimum is in 4th quadrant

$\begin{array}{c|c} 2 & 1 \\ \hline 3 & 4 \end{array}$

ANSWER:



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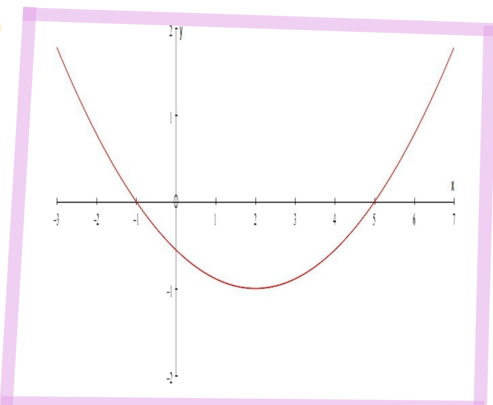

Q6

(Total for Question 6 is 7 marks)

7. A curve with equation $y = f(x)$ passes through the point (4, 25)

Given that

$$f'(x) = \frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1, \qquad x > 0$$

find f(x), simplifying each term.

(5)

$f(x) \underset{\text{integrate}}{\overset{\text{differentiate}}{\rightleftarrows}} f'(x)$

① Integration

$\because x^0 = 1$

$f(x) = \int f'(x)\,dx = \int \frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1x^0\,dx$

$= \left[\left(\frac{3/8}{2+1}x^{2+1}\right) + \left(\frac{-10}{-\frac{1}{2}+1}x^{-\frac{1}{2}+1}\right) + \left(\frac{1}{0+1}x^{0+1}\right)\right] = \frac{1}{8}x^3 - 20x^{1/2} + x + c$

② find +c by substituting (4, 25) so that f(4) = 25.

$f(x) = \frac{1}{8}x^3 - 20x^{1/2} + x + c$

$f(4) = \frac{1}{8}(4)^3 - 20(4)^{\frac{1}{2}} + (4) + c = 25$

$8 - 40 + 4 + c = 25$

$-28 + c = 25$

$+28 \qquad c = 53 \qquad +28$

$\therefore f(x) = \frac{1}{8}x^3 - 20x^{\frac{1}{2}} + x + 53$

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Question 7 continued

Q7

(Total for Question 7 is 5 marks)

8.

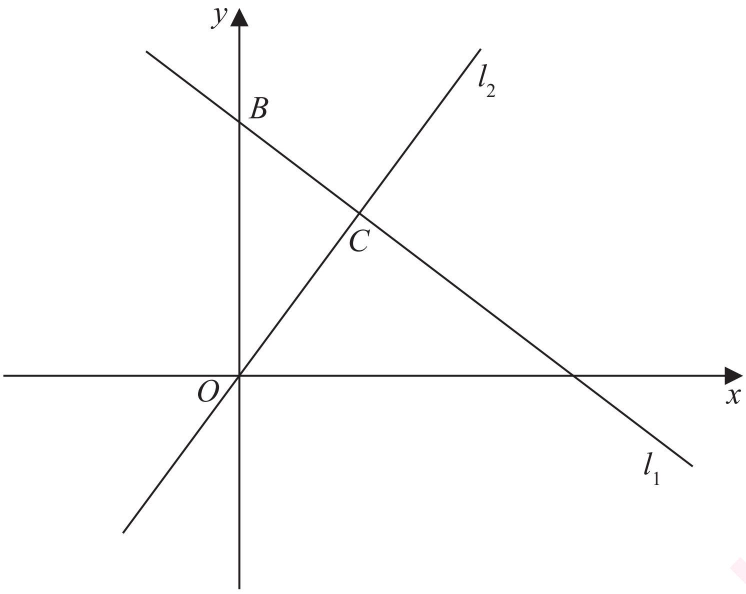


Figure 2

The line l_1, shown in Figure 2 has equation $2x + 3y = 26$

The line l_2 passes through the origin O and is perpendicular to l_1

(a) Find an equation for the line l_2

(4)

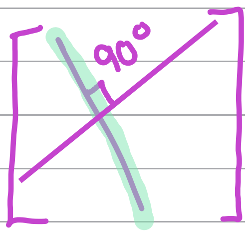
The line l_2 intersects the line l_1 at the point C. Line l_1 crosses the y-axis at the point B as shown in Figure 2.

(b) Find the area of triangle OBC. Give your answer in the form $\frac{a}{b}$, where a and b are integers to be found.

(6)

a) l_2 is perpendicular to l_1. l_2 is normal to l_1.

normal is Perpendicular to line



∴ we find gradient of normal (m_n)

using Perpendicular gradient rule $m_{normal} \times m_{line} = -1$

① find gradient of l_1 by rearranging equation into the form $y = mx + c$ where m is gradient.

$l_1: 2x + 3y = 26$

$-2x$ ($3y = -2x + 26$) $-2x$

$\div 3$ ($y = -\frac{2}{3}x + 26$) $\div 3$ → $\boxed{\begin{matrix} y = mx + c \\ y = -\frac{2}{3}x + c \end{matrix}}$

Question 8 continued

② find gradient of l_2 normal using perpendicular gradient rule.

$$m_n \times m_{l_1} = -1$$
$$\div -\tfrac{2}{3} \quad m_n \times -\tfrac{2}{3} = -1 \quad \div -\tfrac{2}{3}$$
$$m_n = \frac{3}{2}$$

③ Find equation of line l_2 using line passing through (a, b) & gradient m.

$$\text{equation: } (y - b) = m(x - a)$$

l_2 passes through Origin which is $(0, 0)$

$a = 0$
$b = 0$
$m = 3/2$

$$(y - 0) = \frac{3}{2}(x - 0)$$

$$\therefore \; l_2 : \; y = \frac{3}{2}x$$

b) Triangle OBC. O is origin $(0, 0)$

① Find point C by substituting l_2 into l_1 to find point of intersection (C).

$$l_1 : 2x + 3y = 26$$
$$l_2 : y = \frac{3}{2}x$$

$$2x + 3y = 26$$
$$2x + 3\left(\frac{3}{2}x\right) = 26$$
$$2x + \frac{9}{2}x = 26$$
$$\div \tfrac{13}{2} \quad \frac{13}{2}x = 26 \quad \div \tfrac{13}{2}$$
$$x = 4$$

Question 8 continued

Substitute $x = 4$ into l_1 or l_2 to find y-coordinate.

$$l_2 : \; y = \frac{3}{2}x$$
$$y = \frac{3}{2}(4)$$
$$y = 6$$

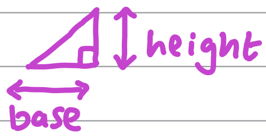
$$\therefore \; C(4, 6)$$

② Find point B by substituting $x = 0$ into equation l_1, as B is where l_1 crosses y-axis (at y-axis, $x = 0$).

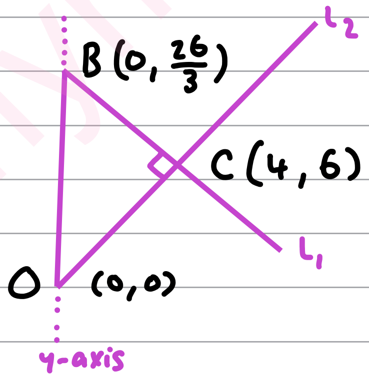
$$l_1 : \; 2x + 3y = 26$$
$$2(0) + 3y = 26$$
$$\div 3 \quad 3y = 26 \quad \div 3$$
$$y = \frac{26}{3}$$

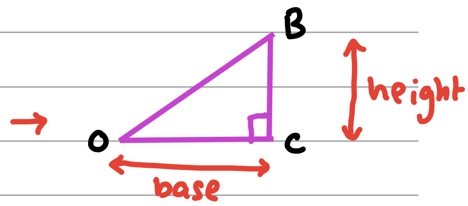
$$\therefore \; B\left(0, \frac{26}{3}\right)$$

④ Find length of base & height of triangle OBC.



Triangle OBC :





height (h) = BC
base (b) = OC

length between 2 points : $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

B $\left(0, \; ^{26}/_{3}\right)$
C $(4, \; 6)$

$$|BC| = \sqrt{(0-4)^2 + \left(^{26}/_{3} - 6\right)^2}$$
$$= \frac{4\sqrt{13}}{3}$$

Question 8 continued

$O\ (0\ ,0)$ $\quad |OC| = \sqrt{(0-4)^2 + (0-6)^2}$

$C\ (4\ ,6)$ $\quad = \sqrt{52} = 2\sqrt{13}$

⑤ Find Area of OBC using Area of a triangle : $A = \frac{1}{2}bh$

$$A_{OBC} = \frac{1}{2} \times 2\sqrt{13} \times \frac{4\sqrt{13}}{3} = \frac{52}{3}$$

$\therefore$ Area of triangle OBC $= \frac{52}{3}$ units^2 $\quad a = 52 \quad b = 3$

Leave blank

Q8

(Total for Question 8 is 10 marks)

9.

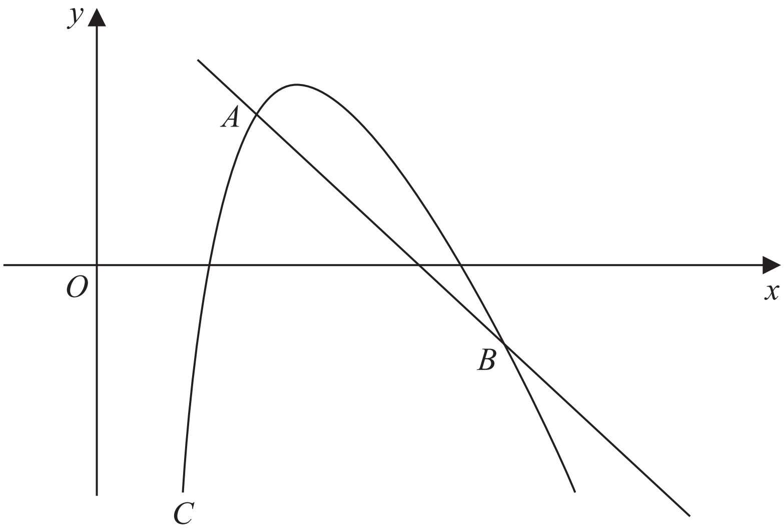


Figure 3

A sketch of part of the curve *C* with equation

$$y = 20 - 4x - \frac{18}{x}, \qquad x > 0$$

is shown in Figure 3.

Point *A* lies on *C* and has *x* coordinate equal to 2

(a) Show that the equation of the normal to *C* at *A* is $y = -2x + 7$.

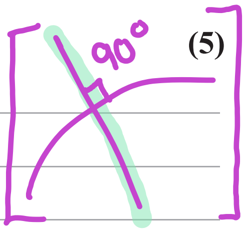
(6)

The normal to *C* at *A* meets *C* again at the point *B*, as shown in Figure 3.

(b) Use algebra to find the coordinates of *B*.

(5)

a) normal is Perpendicular to Curve



∴ we find gradient of normal (m_n)

using Perpendicular gradient rule $m_{normal} \times m_{curve} = -1$

① Find gradient function (differential) of Curve C.

C: $y = 20 - 4x - \frac{18}{x^1}$ * $= 20 - 4x - 18x^{-1}$

* indicies rule : $\frac{a}{x^b} = ax^{-b}$

C: $y = 20x^0 - 4x^1 - 18x^{-1}$ · $x^0 = 1$

$dy/dx = 0(20x^{0-1}) + 1(-4x^{1-1}) + (-1)(-18x^{-1-1})$

$= -4 + 18x^{-2}$

Question 9 continued

② find gradient of curve at $x = 2$.

$$\left.\frac{dy}{dx}\right|_{x=2} = -4 + 18(2)^{-2} = \frac{1}{2}$$

③ find gradient of normal using formula.

$$M_n \times M_c = -1$$

$$\div \tfrac{1}{2} \quad M_n \times \tfrac{1}{2} = -1 \quad \div \tfrac{1}{2}$$

$$M_n = -2$$

④ Find coordinates of point where normal meets curve at $x = 2$.

$$C: \quad y = 20 - 4(2) - \frac{18}{(2)} = 3$$

$$\therefore (2, 3)$$

⑤ Find equation of normal using line that passes through (a, b) & gradient M.

$$\text{equation: } (y - b) = M(x - a)$$

$a = 2$
$b = 3$
$M = -2$

$$(y - 3) = -2(x - 2)$$

⑥ rewrite in form $y = mx + c$

$$y - 3 = -2(x - 2)$$

$$+3 \quad y - 3 = -2x + 4 \quad +3$$

$$y = -2x + 7$$

$$\therefore y = -2x + 7$$

Leave blank

Question 9 continued

b) ① equate equation of normal with equation of curve C.

normal : C

$$-2x + 7 = 20 - 4x - \frac{18}{x}$$

$+2x$ $+2x$

$$7 = 20 - 2x - \frac{18}{x}$$

-7 -7

$$0 = 13 - 2x - \frac{18}{x}$$

$\times x$ $\times x$

Forming a quadratic : $0 = 13x - 2x^2 - 18$

$\times -1$ $\times -1$

$$0 = -13x + 2x^2 + 18$$

② Solve for x using quadratic formed

$$2x^2 - 13x + 18 = 0$$

factorise : $(2x-9)(x-2)=0$

Solve : • $2x - 9 = 0$ • $x - 2 = 0$

$2x = 9$ $\therefore x = 2$

$\therefore x = \frac{9}{2}$

Second point where C & normal meet ∴ this is x-coordinate of B.

this is x-coordinate of first point where C & normal meet in part (a).

③ Find y-coordinate of B by Substituting $x = \frac{9}{2}$ into equation C or normal.

normal: $y = -2x + 7$

$$y = -2\left(\frac{9}{2}\right) + 7$$

$$y = -9 + 7$$

$$\therefore y = -2$$

$$\therefore B\left(\frac{9}{2}, -2\right)$$

Q9

(Total for Question 9 is 11 marks)

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DO NOT WRITE IN THIS AREA

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10.

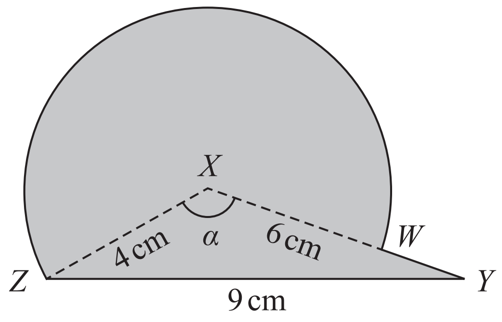


Figure 4

The triangle XYZ in Figure 4 has $XY = 6$ cm, $YZ = 9$ cm, $ZX = 4$ cm and angle $ZXY = \alpha$.

The point W lies on the line XY.

The circular arc ZW, in Figure 4, is a major arc of the circle with centre X and radius 4 cm.

(a) Show that, to 3 significant figures, $\alpha = 2.22$ radians.

↳ UNITS !!

(2)

(b) Find the area, in cm^2, of the major sector $XZWX$.

(3)

The region, shown shaded in Figure 4, is to be used as a design for a logo.

Calculate

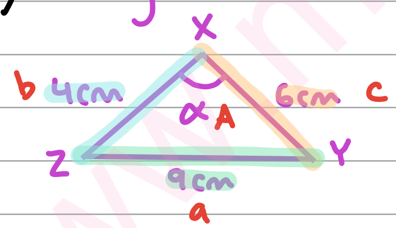
(c) the area of the logo

(3)

(d) the perimeter of the logo.

(4)

a) Using Cosine rule



Pure Mathematics P1

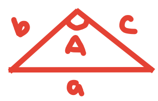
Mensuration

Surface area of sphere $= 4\pi r^2$

Area of curved surface of cone $= \pi r \times$ slant height

Cosine rule

$a^2 = b^2 + c^2 - 2bc\cos A$



$$ZY^2 = XZ^2 + XY^2 - 2(XZ)(XY)\cos\alpha$$

$$9^2 = 4^2 + 6^2 - 2(4)(6)\cos\alpha$$

$$81 = 16 + 36 - 48\cos\alpha$$

$$81 = 52 - 48\cos\alpha \quad -52$$

$$-52 \quad 29 = -48\cos\alpha \quad \div -48$$

$$\div -48 \quad -\frac{29}{48} = \cos\alpha$$

Question 10 continued

inverse cosine $\cos\alpha = -\dfrac{29}{48}$ inverse cosine

$\alpha = \cos^{-1}\left(-\,{}^{29}\!/_{48}\right)$

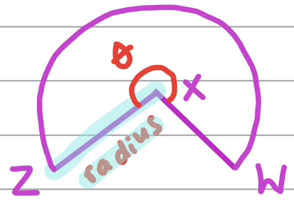
$\alpha = 2.21951..$

≈ 2.22 radians

$\therefore\ \alpha = 2.22$ rad (3 s.f.)

b) Area of sector XZWX. Area of sector : $A = \frac{1}{2}r^2\theta$

Sector XZWX :



radius $= ZX = XW = 4$cm

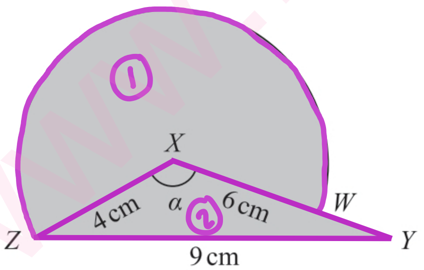
θ = obtuse $\angle ZXW = \underbrace{2\pi}_{\text{Angle in a circle is } 2\pi\ (360^\circ)} - \underbrace{2.22}_{\alpha\ /\ \text{acute} \angle ZXW \text{ from part (a)}}$

$\therefore\ A = \frac{1}{2} \times (4)^2 \times (2\pi - 2.22) = 32.50548... \approx 32.5$ (3 s.f.)

$\therefore$ Area of Sector XZWX $= 32.5\text{ cm}^2$

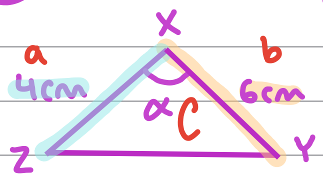
c) Total Area = Area sector XZWX + Area triangle ZXY

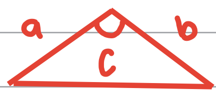
= 1 + 2



1 $= 32.5\text{ cm}^2$ from part (b)

2 Find area of triangle XYZ. Area of triangle : $A = \frac{1}{2}ab\sin C$





$a = XZ = 4$ cm

$b = XY = 6$ cm

$C = \angle ZXY = \alpha = 2.22$ rad

Question 10 continued

$A = \frac{1}{2}(4)(6)\sin(2.22)$

$= 9.55878..$

$\approx 9.56 \ (3\text{s.f.})$

③ Total Area $= ① + ②$

$= 32.5 + 9.56$

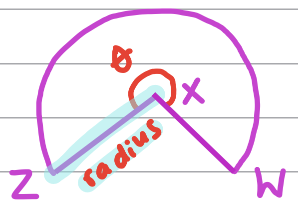
$= 42.06$

$\approx 42.1 \ (3\text{s.f.})$

$\therefore$ Area of logo $= 42.1 \text{ cm}^2$

d) Perimeter = arc ZW + WY + YZ

① length of arc ZW length of arc : $S = r\theta$



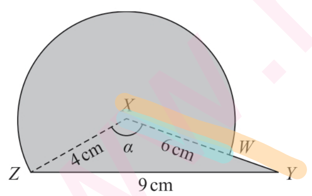
$r = XZ = XW = 4\text{cm}$

$\theta =$ obtuse $\angle ZXW = (2\pi - 2.22)$ rad

$S = 4 \times (2\pi - 2.22) = 16.25274..$

$\approx \boxed{16.3} \ (3\text{sf})$

② find length WY.



XW is a radius $\therefore$ XW = 4cm.

$XWY = XW + WY$

$6 = 4 + WY$

$-4 \quad 2 = WY \quad -4$

$\therefore WY = \boxed{2}$

③ $YZ = \boxed{9}$

Question 10 continued

(4) Perimeter = (1) + (2) + (3)

$= 16.3 + 2 + 9$

$= 27.3$

$\therefore$ Perimeter of logo $= 27.3$ cm

Q10

(Total for Question 10 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS